Wavelet Based Compressive Sensing Techniques for Image Compression

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ABSTRACT

Compressive sensing (CS) exploits the sparsity of the commonly encountered signals and provides the data compression at the first step of the image acquisition. In this paper, performance of various wavelet based CS techniques has been analysed. It is based on the concept that small collections of non-adaptive linear projections of a sparse signal contain enough information for its effective reconstruction using some optimization procedure. Wavelet Transform is widely applied to the domain of CS to obtain the sparse representation of the signals to be compressed. The results of CS techniques prove that the image reconstruction quality obtained by wavelet based CS techniques is better than the practical image compression standards like JPEG.

Keywords: **Compressive Sensing, Contourlet Transform, Slepian Wolf Coding (SW), Wavelet Transform (WT) .**

1. Introduction

Compressive sensing (CS) is a new approach to simultaneous sensing and compression that enables a potentially large reduction in sampling and computation costs at a sensor for signals having a sparse representation in some basis. Generally, to avoid losing information when capturing a signal, one must sample at least two times faster than signal bandwidth (Nyquist Sampling Theorem). But in some applications, Nyquist rate is very high, which results in the larger number of samples and hence their compression become a necessity before their storage and transmission.

CS can be applied effectively to the sparse signals but most of the signals encountered practically are not sparse. So either Discrete Cosine Transform (DCT) or Wavelet transform (WT) is first applied on the signals to get their sparse representation. Wavelet transform based CS techniques are more successful as the wavelets have time-frequency location and multi-resolution characteristics , so it can decompose the image signal into number of sub-band signals which are with different spatial resolution, frequency and directional characteristics. Hence, the low frequency characteristics of long and high frequency characteristics of short features can be dealt simultaneously. Also Wavelet transform can overcome the block artifacts introduced in the reconstructed image by the DCT. The main reason of moving on to Compressive Sensing is due to three significant insufficiencies suffered by the widespread used transform coding. Firstly, the no. of samples (N), which are initially taken, may be very large even if the desired number of samples (K) is small. Secondly, the set of transform coefficients for all the N samples i.e (Si) must be computed even though only desired K sample coefficients are retained and rest all are discarded. Thirdly, the location of K large co-efficient must be encoded thus introducing an overhead.

In the following section, background of compressive sensing and wavelet transform has been discussed. In Section 3, various wavelet based compressive sensing techniques have been discussed. Section 4 contains results and in last Section 5 the paper has been concluded.

2. Compressive Sensing and Wavelet Transform

Compressive Sensing can be applied on 1-D finite signals. To understand the concept of compressive sensing, let a finite length, real valued, one - dimensional, discrete time signal x has been considered. Here the function x being expanded is discrete, so the resulting coefficients are called *discrete wavelet transform* (DWT) obtained by the use of a set of Fourier basis function Ψ. For simplicity it is assumed that these basis Ψ are orthonormal. Signal x can be sparsely represented using Fourier basis Ψ.

$$x=\sum_{i=1}^{N} Si\Psi i \text{ or } x=s\,\Psi \qquad (1)$$

'Si' is sparse signal matrix with K nonzero coefficients and it is (N*1) column vector. The signal x is K sparse if K of the Si coefficients are nonzero and (N-K) are zero and (K<<N). Signal x is compressible if it has a few large coefficients and many small coefficients. Now general linear measurement is done that computes M<N inner products between x and a collection of vectors $\Phi_{j=1}^{M}$ as in, $y_j=\langle x * \Phi j\rangle$ as in [1].The signal x can be recovered from its measurements when the measurement matrix Φ is incoherent with the dictionary Ψ on which the signal is sparse over. Some choices for the measurement matrix, Φ which satisfies RIP are a random matrix with i.i.d. Gaussian entries or the Bernoulli (±1) matrix . Using such a matrix Φ of size cK × N, where c is an oversampling factor, it was shown that it is possible, with high probability, to recover any signal that is K-sparse in the dictionary Ψ from its projection onto Φ.

Given the M vector of measurements, [y=Φx], the recovery algorithm consists of solving the convex problem

$$min_{x'}\|\Psi * x'\|_{l1} \quad \text{subject to} \quad \Phi x' = y \qquad (2)$$

Basically l_1 minimization norm is used for reconstruction because (i) Sparse signals have small l_1 norms related to their energy. (ii) It is convex, which makes optimization problem tractable.

$$\hat{s} = \text{argmin} \|s'\|_1 \quad \text{such that } \Theta s' = y. \qquad (3)$$

where as $\hat{s}$ is the approximation of the original signal x. The location of the important transform coefficients can be determined and their value can be reconstructed from (3). The importance of knowing the location of important transform coefficients comes from the fact that the image that is acquired is sparse in the wavelet domain but spread in the measurement domain, so the location of important values are not known. The (3) 'triangulates' the locations of important transform coefficients and their values.

3. Analysis of Wavelet Based Compressive Sensing Techniques

When the image data is sent over noisy channel which provides a high risk of bit loss then CS helps in retrieval of the lost information as CS measurements of 2D-DWT carry nearly the same amount of information and hence bit loss effects get reduced where as in all widely used image compression standards like JPEG, JPEG-2000, only channel coding deals with this issue.

In this scheme, the original image is first normalized to (0-1) at the encoder then DWT is performed to get its sparse representation as in [2]. Based on the properties of the 2D DWT, these coefficients are re-sampled using measurement matrix Φ and then resultant random measurements are quantized. For the collection of measurements previously used W-CS was replaced by M-CS as it gives better results than (W-CS) and in each M-CS sub-band can be grouped and measured as follows:-

$$\begin{matrix} Y(1) \\ Y(2) \\ Y(3) \end{matrix} = \begin{matrix} \Phi(1) & 0 & 0 \\ 0 & \Phi(2) & 0 \\ 0 & 0 & \Phi(L) \end{matrix} \begin{matrix} X(1) \\ X(2) \\ X(L) \end{matrix} \qquad (4)$$

$$\text{Yi} = [y_{i,1}\, y_{i,2} \ldots\ldots y_{i,Mi}] \qquad (5)$$

$$\text{Xi} = [x_{i,1}\, x_{i,2} \ldots\ldots x_{i,Ni}] \qquad (6)$$

Yi and Xi denote the compressive sensing measurements and wavelet coefficients respectively at decomposition level i =[i= 1,2….L]. Mi and Ni represent the number of samples and coefficients at the corresponding level. Φi represents the Gaussian random matrices applied for compressive sensing measurements. In this scheme, the criteria for measurement allocation was to include all the measurements from the decomposition level L which has small number of coefficients carrying major information about the original image and for the sub-band in other scales, the CS measurements are distributed according to the bit budget and the number of coefficients at different levels. Before sending to decoder, the data is packeted and to each packet a prefix is added so that the decoder can know which packets have been received. The packets that are not been received, their corresponding rows in the measurement matrix are deleted and compressive sensing reconstruction is done by solving the following l_1 minimization problem

$$f^l = \text{arg min}\| w^l \Psi^* \|_{l1} \text{ subject to } \|\Phi f\_y\|_{l2} \leq \varepsilon \qquad (7)$$

where Ɛ is the error bound for the effective reconstruction and l_1 and l_2 represents minimization procedures.

In remote sensing systems which face the problem of limited computational source and transmission bandwidth ,wavelet based CS is combined with Vector Quantization (VQ) and Arithmetic Coding (AC) as in [3]. In this technique, DWT is performed to get the sparse representation of the image then linear projections of the sensed image are measured at the transmitting side, these measurements are then represented in the terms of indices of the codebook which are further subjected to AC for the purpose of bit reduction (in which more frequently used indices are represented by few bits and rarely used indices are represented by large bits) To know about the occurrence of indices, their probabilities of occurrence should be known which is obtained by studying the histograms of set of VQ training images. Then the variable numbers of bits for the compressed and quantized measurements due to the Arithmetic Coding of the indices are sent to the ground station of remote sensing system. At the decoder, received AC indices are decoded using same histograms as in encoder. Then the codebook indices are decoded using VQ look-up tables to get the CS measurements from which then the reconstructed image is formed. The complete scheme has been diagrammatically depicted in "Fig. 1".

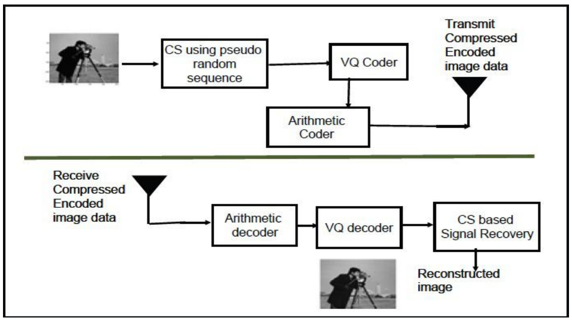


Figure1. The block diagram of transmit and receive side of wavelet based CS technique combined with VQ and AC.

In Compressive Sensing of color images, the image is acquired in the form of three channels i.e. red (R), green (G), blue (B) channels which have sparse representation in Wavelet domain as in [4]. The equation of this transformation can be shown as:

$$C=\Phi^T x_c,\ C \in [R, G, B],\ x_c=\Phi C \qquad (7)$$

where C represents each channel, x_c represents sparsely transformed coefficients of color channel and Φ represents transformation matrix. The lower dimensional projection of each color signal is collected as:

$$y_c = A_c C = A_c\Phi^T x_c,\ ,\ C \in [R,G,B] \qquad (8)$$

These three channels are highly correlated as the sparse transformed coefficients of all the channels have been obtained by orthogonal transformation (Ψ is ortho-normal base function), so if the value at index j is high in x_R , then it will also be high in x_G and x_B.In the image reconstruction by convex optimization, only one of the coefficients from the group of correlated coefficients are likely to be selected, but to get the better result, *non-convex group sparse optimization* is preferred which guarantee the selection of all the three high valued coefficients at the same index. Mathematically, it involves solving the problem

$$\min \|x\|_{2,p} \text{ subject to } y=Mx. \qquad (9)$$

Compressive Sensing can be applied to 1-D data so when CS is applied to 2-D contourlet sub-band , the sub-band needs to be converted to 1-D long vector which is then measured by large random matrix as in [5] and hence the number of computations and the storage space requirement is very high, so the application of CS reconstruction process to sparse contourlet sub-bands is avoided. In this scheme, transformation of contourlet sub-bands are done in wavelet domain in order to concentrate the sub-bands in small subspaces and then these generated wavelet coefficients are used for CS framework which then result in reduction in size of measurement matrix Φ. At the decoder, inverse wavelet transform is applied on the received signal. As signal was transmitted over noisy channel so thresholding is done to denoise the image. After this inverse contourlet transform is implemented, this is then finally subjected to Wiener filtering to get the reconstructed imagex_{rec}. The proceedings at the encoder and the decoder have been shown below in "Fig. 2".

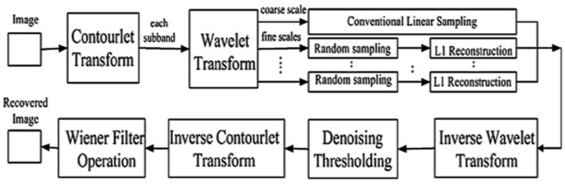


Figure2.The flowchart of CS reconstruction based on wavelet in the contourlet domain

Wavelet based Compressive Sensing is also used in medical imaging field like for ECG and EEG signal processing as in [6]. First, to get the sparse data from the ECG and EEG signals combined scheme of wavelet transform (Daubechies Wavelet) and iterative thresholding is used which gives a good performance of denoising on the data. Then the sparsely represented data is fed to CS framework for further compression. CS measurement can be represented as

$$G=\lambda w=\lambda w_s+\lambda w_e= \lambda w_s+n_e \qquad (10)$$

w_s represent an *N*-dimensions vector that is identical to vector w, which has *M* elements with the largest magnitude w_e is identical to w for the smallest *N-M* elements . n_e are approximated as a zero-mean Gaussian noise with unknown variance σ^2. In the case of ECG signals, QRS wave carries major information about state of heart, so prior to compression QRS wave is extracted from ECG signals, but when only QRS wave is considered wavelet transform is not used. Then at the receiving station, basic pursuit (BP) or Bayesian compressive sensing (BCS) algorithm is used for reconstruction from the received signals.

Radio Astronomy makes use of the CS as the observations of the sky using interferometers are inherently under-sampled in angular frequency as in [7]. This scheme helps to extract information on particular angular scales but limits the fidelity of the reconstructed images as well as their dynamic range, and consequently significant effort and expertise is invested in optimal image reconstruction techniques. The original sky signal is first sparsely represented by wavelet transform using Fourier basis Ψ, then the obtained wavelet coefficients are multiplied by the aperture illumination function of our telescope as x.When observed with an interferometer the signal is sampled using the noisy Fourier basis Φ.

$$y = M\,\Phi\, x + n \qquad (11)$$

Due to the finite number of baselines in our interferometer, the sampling is incomplete in Fourier space and this is included through the matrix M. ni is the Gaussian noise on a measured visibility and the sampled measurements, yi, are simply the recovered visibilities. At the ground station, the CS reconstruction procedure can be applied to get the recovered signal x* from y. There is one limitation for use of this method for radio interferometry as the sampling distribution produced by interferometers is deterministic in Fourier space but CS is optimally performed by random sampling of Fourier frequencies. However this problem may be tackled by re-weighting the visibility distribution in the same way as is done to improve the form of the synthesized beam using so-called normal weighting, which homogenizes the spatial weight of the distribution of visibilities in uv space.

4. Results

The results obtained from above discussed techniques make CS applicable in different areas. In the first technique of CS, extensive experimental results have been reported to verify the robustness of the CS-based image coding scheme with packet loss. Also the results show the impact of the choice of wavelet transform on the reconstructed image. 512 × 512, 8 bpp (bit per pixel) greyscale image Lena, were used for evaluation. Test image was encoded at 1.0 bpp and 0.5 bpp, the packeted bit stream was sent through a noisy binary symmetric channel (BSC) channel under various packet loss rates (PLRs) (up to 0.5).It can seen that the CS measurements architecture (M-CS) is modified from the conventional wavelet CS scheme (W-CS) in which measurement from each different scale is collected as per the number of wavelet coefficient at the corresponding level (used in [8]). So, the main difference between W-CS and M-CS lies in the CS measurements allocation “Fig. 3”shows that M-CS outperforms W-CS as in [2].

(a)

(b)

Figure3. showing performance for 512×512 Lena coded at 1.0bpp with PLR=0.3 (a) W-CS; (b) M-CS

When CS is used in combination with VQ and AC for remote sensing systems, the results can be evaluated using MATLAB. Here, to show the performance of this algorithm a man’s image of size 1024x1024 pixels has been taken. This image is segmented into blocks of size 128 x128 and then CS framework has been applied at both the transmitting and receiving side. The results that are obtained are then compared with DCT based JPEG in which image is segmented into blocks of size 8 x8 pixels. However, PSNR of JPEG recovered image is slightly higher but the computation time in CS scheme at the encoder has 110x reduction in latency on the transmit side as compared to JPEG. Moreover, the reconstruction quality of reconstructed image obtained by CS technique is better than that of JPEG as in [3] as shown in “Fig. 4”.

(a)

(b)

(c)

Figure4. Original image and Comparison between CS and JPEG reconstructed images at the 0.47 and 0.5 bpp, respectively.

In CS based color image compression, the reconstructed image can be obtained by convex optimization or by non convex group sparse optimization. Both differ in the way in which the CS measure correlated coefficients are considered either in group or individually respectively. The experimental results show that the later technique gives better results. As when the image of Barbara, Lena are reconstructed by both convex and non-convex group optimization (by l_2 minimization and p=0.4) at projection function 10, the PSNR obtained is between (24 dB-26.1dB) and (26.30dB-27.60dB) respectively as in [4] shown below in “Fig. 5”.

Figure5. (L–R) Barbara,Lena.

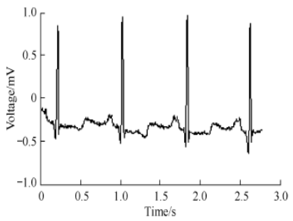
When only Wavelet Transform is used in CS reconstruction, the computation burden involved is comparably small but the quality of reconstruction is much better when the CS based on Wavelet Transform in Contourlet domain is performed as wavelets could not provide the sparsest representation when applied in isolation. The difference in the results produced by the two schemes as in [5] can be clearly seen in the “Fig. 6”.

(a)

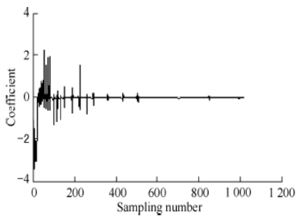
(b)

Figure6. Recovery results of image'Lena':(a) reconstruction from CS based on wavelet [2] by BP, S_{max}=1536_3072, PSNR=29.437c)reconstruction from CS based on the wavelet transform in contourlet domain by BP, S_{max}=1536_3072, PSNR=36.194.

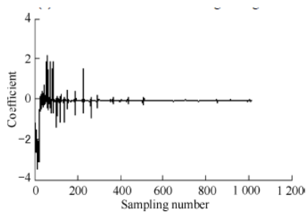
Now , the results obtained by ECG signal processing using CS technique has been shown from which it is clear that after wavelet transform the signal turns into sparse signal then compressive sensing is executed to the sparse coefficients of the signals with iterative threshold method. Next, the BCS reconstructing method is used to reconstruct the coefficients as shown in"Fig. 7(c)". Finally, the original signal can be restored as shown in "Fig. 7(d)" by the reconstructed coefficients as in [6] by using inverse wavelet transform.



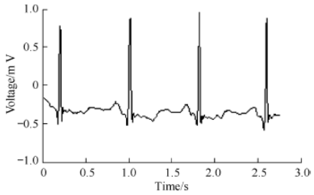
(a)



(b)



(c)



(d)

Figure7. ECG signals with BCS (a) Original signal (b) Wavelet coefficients of the original signal (c) Reconstructed coefficients (d) Reconstructed signal

5. Conclusion

From above results it can be concluded that CS has potential to replace or stand among the other image compression standard widely in use (JPEG and JPEG-2000). The results which are obtained are better than the linear reconstruction techniques. Also it provides the additional advantage of robust image compression which is very important as most of the signals /images are practically sent over noisy channel. Wavelet based CS techniques prove better than DCT based JPEG or DCT based CS as the parallel computing can't be performed on the image transformed by using DCT due to the introduction of block artifacts in the reconstructed image. So the system which are limited by the battery power, wavelet based CS has the computational advantage over JPEG when low power parallel processors such as micro-core parallel processors are used.

CS holds significant unrealized potential for application to highly detailed natural images, particularly in environment characterized by moderate to high values of high frequency information and/or noise, the potential for missing image data, and asymmetrical communication systems. But the main limitation of CS is that it can be effectively applied for the compression of high frequency information and the significant amount of attention that has been given to the theoretical aspects of compressive sensing, the practical image compression is still dominated by JPEG and JPEG-2000.

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